Normativity of logic and change of subject

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Normativity of logic and right logic

Normativity of logic

Subjective normativity of logic: If we believe $A \models B$ and we believe A, we ought to believe B;

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Objective normativity of logic: Subjective normativity of logic + we ought to believe in **the right logic**.

We need logical disagreement!

Normativity of directives, of evaluations and of appraisals in (Steinberger, *Three Ways in Which Logic Might Be Normative*, Journal of Philosophy (116), 2019, pp. 5-31.)

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Classical logic: $\models p \vee \neg p$

Intuitionistic logic: $\not\models p \vee \neg p$

In order to disagree, **two logics have to speak about the same logical terms.**

They disagree

- $\models_k p \vee \neg p$
- $\not\models_i p \vee \neg p$

(homophonic translation)

They don't disagree

- $\models_k p \vee_k \neg_k p$
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Two logical terms are the same if they express the same meaning.

	Sentence	Logical Constant
Realism	Truth conditions	Truth tables
Antirealism	Assertion conditions	Inference rules

$$\begin{tikzcd}
 & & \text{same truth table} \arrow[dr] & \\
\text{identity} \arrow[r] & \text{same meaning} \arrow[ur] \arrow[dr] & & \text{same laws} \\
 & & \text{same rules} \arrow[ur] &
\end{tikzcd}$$

if they are the same, they validate the same logical laws.

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if they are the same, they validate the same logical laws.

1. **In order to disagree, two logics have to speak about the same logical terms;**
2. **If two logical terms are the same then they validate the same logical laws**.

So logical disagreement seems to be impossible and there is nothing like the true logic!

'Change of logic, change of subject.' (Quine, *Philosophy of Logic*, 1986, chapter *Deviant logics*.)

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A realist solution

Meaning and behaviour in realism

$A \vee B$	T	F
T	T	T
F	T	F

$A \vee B$	T	U	F
T	T	T	T
U	T	U	U
F	T	U	F

Idea: **K** is essentially **K**$_3$ where there are no gaps!

Realist meaning + **metaphysical shape of the models** → **behaviour**.
In general:

Generalized Tarski Thesis (GTT): An argument is **valid**$_x$ if and only if, in every **case**$_x$ in which the premises are true, so is the conclusion.(Beall & Restall, *Logical Pluralism*, 2006, p. 29.)

Different sets of cases detect different logics: cases can have gaps in truth values, gluts in truth values, etc.

A logical realist can reject the point **2** of the puzzle: **two logical terms can be the same also if they validate different logical laws.**

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An antirealist solution

Two factions in proof-theoretic semantics:

Prawitz, Dummett & Steinberger: $\not\models p \vee \neg p$

Boričić, Read, Milne & Rumfitt: $\models p \vee \neg p$

We will see that **their disagreement is not only apparent!**
They don't talk past each other.

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Natural deduction as a **theory of meaning**.

- **Introduction** rules are **meaning conferring**;
- **Elimination** rules are justified if, when we have the **major premise of an E-rule derived using an I-rule**, then we can **reduce the proof**;
- Otherwise the E-rules are not justified!

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Harmony of $\vee$

$$
\dfrac{\dfrac{\begin{array}{c}\vdots\Phi_1\\ A\end{array}}{A \vee B}\,\vee I \qquad \begin{array}{c}[A]\\ \vdots\Phi_2\\ C\end{array} \qquad \begin{array}{c}[B]\\ \vdots\Phi_3\\ C\end{array}}{\begin{array}{c}C\\ \vdots\Phi_4\end{array}}\,\vee E
\qquad \rightsquigarrow \qquad
\begin{array}{c}\vdots\Phi_1\\ [A]\\ \vdots\Phi_2\\ C\\ \vdots\Phi_4\end{array}
$$

$$\begin{array}{c}\dfrac{\dfrac{\begin{array}{c}\vdots\,\Phi_1\\ A\end{array}}{A \vee B}\,\vee I \qquad \begin{array}{c}[A]\\ \vdots\,\Phi_2\\ C\end{array} \qquad \begin{array}{c}[B]\\ \vdots\,\Phi_3\\ C\end{array}}{C}\,\vee E\\ \vdots\,\Phi_4\end{array} \qquad \rightsquigarrow \qquad \begin{array}{c}\vdots\,\Phi_1\\ [A]\\ \vdots\,\Phi_2\\ C\\ \vdots\,\Phi_4\end{array}$$

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C\\
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Prior's tonk

$$Itonk\ \frac{A}{A tonk B} \qquad Etonk\ \frac{A tonk B}{B}$$

$$\begin{array}{r}Itonk\\Etonk\end{array}\ \dfrac{\dfrac{A}{A tonk B}}{B} \quad \rightsquigarrow \quad ?$$

- *Tonk* is not an harmonious connective, and indeed it leads to triviality in standard logical systems.

Prior's tonk

$$Itonk \ \frac{A}{AtonkB} \qquad\qquad Etonk \ \frac{AtonkB}{B}$$

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$$\frac{A}{A \vee B} \vee I \qquad \frac{A \vee B \quad \begin{matrix}[A] \\ \vdots \\ C\end{matrix} \quad \begin{matrix}[B] \\ \vdots \\ C\end{matrix}}{C} \vee E$$

$$\frac{B}{A \vee B} \vee I \qquad \frac{A \supset B \quad A}{B} \supset E$$

$$\frac{\begin{matrix}[A] \\ \vdots \\ B\end{matrix}}{A \supset B} \supset I$$

$$\frac{\bot}{A} \bot E$$

Conjecture (Prawitz & Dummett): **We can not prove *tertium non datur* using harmonious rules!**

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Solutions for *tertium non datur*

- **Boričić & Read: multiple conclusions!**

$$\dfrac{\dfrac{[A]^1}{A, \bot}\ \text{Weakening}}{A, A \supset \bot} \supset I_1$$

- **Milne: weakening of separability!**

$$I \supset_{Mln} \dfrac{\begin{matrix}[A]\\ \vdots \\ B \vee D\end{matrix}}{(A \supset B) \vee D} \qquad I\neg_{Mln} \dfrac{\begin{matrix}[A]\\ \vdots \\ D\end{matrix}}{\neg A \vee D}$$

- **Rumfitt: (bilateralism) assertion and denial!**

$$\dfrac{\dfrac{\dfrac{[-(A \vee \neg A)]^1}{-A} - \vee E \qquad \dfrac{\dfrac{[-(A \vee \neg A)]^1}{-(\neg A)} - \vee E}{+A} - \neg E}{\bot}\ \text{Non-contr}}{+(A \vee \neg A)}\ \text{Reductio}_1$$

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- **Rumfitt: (bilateralism) assertion and denial!**

$$\dfrac{\dfrac{[-(A \vee \neg A)]^1}{-A}\ - \vee E \qquad \dfrac{\dfrac{[-(A \vee \neg A)]^1}{-(\neg A)}\ - \vee E}{+A}\ -\neg E}{\dfrac{\bot}{+(A \vee \neg A)}\ \text{Reductio}_1}\ \text{Non-contr}$$

- **Boričić & Read: multiple conclusions!**

$$\dfrac{\dfrac{[A]^1}{A, \bot}\text{ Weakening}}{A, A \supset \bot}\supset I_1$$

- **Milne: weakening of separability!**

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- **Multiple conclusions are not commonly used!**

  [. . .] the rarity, to the point of extinction, of naturally occurring multiple-conclusion arguments has always been the reason why mainstream logicians have dismissed multiple-conclusion logic as little more than a curiosity. (Rumfitt, Knowledge by deduction, Grazer Philosophische Studien (77), 2008, pp. 61-84)

- **Multiple conclusions lead to non-constructivity!**

  [. . .] this smuggles in non-constructivity through the back door. (Tennant, The Taming of the True, 1997)

- **Multiple conclusions are just disjunction in disguise!**

  [. . .] in a succedent comprising more than one sentence, the sentences are connected disjunctively; and it is not possible to grasp the sense of such a connection otherwise than by learning the meaning of the constant 'or'. (Dummett, Ibidem, p. 187)

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Objections to Milne's solution

- **We loose harmony!**

  Steinberger and Milne about harmony.

$$
\dfrac{\dfrac{\begin{matrix}[A]^1 \\ \vdots\Phi_1 \\ B \vee C\end{matrix}}{(A \supset B) \vee C}\supset I
\qquad
\begin{matrix}\dfrac{[A \supset B]^2 \qquad A}{B}\supset E \\ \vdots\Phi_2 \\ D\end{matrix}
\qquad
\begin{matrix}[C]^2 \\ \vdots\Phi_3 \\ D\end{matrix}}{D}\vee E_2
$$

vs

$$
\dfrac{\dfrac{\begin{matrix}[A]^1 \\ \vdots\Phi_1 \\ B \vee C\end{matrix}}{(A \supset B) \vee C}\supset I
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- **Bilateral harmony does not entail consistency!**

$$I - \bullet\ \frac{+A \qquad -A}{-\bullet} \qquad\qquad I + \bullet\ \frac{+A \qquad -A}{+\bullet}$$

$$E + \bullet\ \frac{+\bullet}{+A} \quad E + \bullet\ \frac{+\bullet}{-A} \quad E - \bullet\ \frac{-\bullet}{+A} \quad E - \bullet\ \frac{-\bullet}{-A}$$

$$\text{Non-contr}\ \dfrac{E - \bullet\ \dfrac{\dfrac{E + \bullet\ \dfrac{[+\bullet]^1}{+A} \qquad E + \bullet\ \dfrac{[+\bullet]^1}{-A}}{-\bullet}\ \text{Reductio}_1}{+A} \qquad E - \bullet\ \dfrac{\dfrac{E + \bullet\ \dfrac{[+\bullet]^1}{+A} \qquad E + \bullet\ \dfrac{[+\bullet]^1}{-A}}{-\bullet}\ \text{Reductio}_1}{-A}}{\perp}$$

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$$\dfrac{\dfrac{E+\bullet\ \dfrac{[+\bullet]^1}{+A} \qquad E+\bullet\ \dfrac{[+\bullet]^1}{-A}}{E-\bullet\ \dfrac{-\bullet}{+A}}\ \text{Reductio}_1 \qquad \dfrac{E+\bullet\ \dfrac{[+\bullet]^1}{+A} \qquad E+\bullet\ \dfrac{[+\bullet]^1}{-A}}{E-\bullet\ \dfrac{-\bullet}{-A}}\ \text{Reductio}_1}{\bot}\ \text{Non-contr}$$

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Antirealist solution

A logical disagreement is just a disagreement about the shape a theory of meaning can have: multiple conclusions, separability, denial, ...

We do not reject any point of the **puzzle**: we slightly **modify point 1** so to **bypass** it!

Two logic**ians** can disagree about the existence of a good theory of meaning for a logical term.

Prawitz, Dummett & Steinberger: Every theory of meaning for classical logic is not harmonious, so the meaning of $\neg_k$ is not well defined ($\not\models p \vee_k \neg_k p$);

Boričić, Read, Milne & Rumfitt: There is a good theory of meaning for classical logic, so $\models p \vee_k \neg_k p$.

They don't talk past each other.

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Factuality of meaning: there has to be exactly one true theory of meaning!

The true theory of meaning gives us the **true logic** (or the **true logics**).

Disagreement + **factuality of meaning** → **objective normativity**.

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Antirealist normativity

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More antirealist normativity?

Not enough normative!

*“[...] **harmony is not enough to guarantee validity.** Harmony ensures that the consequences of an assertion are no more and no less than the meaning encapsulated in the introduction-rule warrants. But that meaning may itself be corrupt.”*

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Tarski: Semantically closed languages are inconsistent!

$$I\bullet\ \frac{\neg\bullet}{\bullet} \qquad\qquad E\bullet\ \frac{\bullet \qquad \begin{matrix}[\neg\bullet]\\ \vdots \\ D\end{matrix}}{D}$$

$$E\neg\ \dfrac{I\neg_2\ \dfrac{E\neg\ \dfrac{E\bullet_1\ \dfrac{[\bullet]^2 \qquad [\neg\bullet]^1}{\neg\bullet} \qquad [\bullet]^2}{\bot}}{\neg\bullet} \qquad I\bullet\ \dfrac{I\neg_2\ \dfrac{E\neg\ \dfrac{E\bullet_1\ \dfrac{[\bullet]^2 \qquad [\neg\bullet]^1}{\neg\bullet} \qquad [\bullet]^2}{\bot}}{\neg\bullet}}{\bullet}}{\bot}$$

Harmony without validity: bullet

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$$
\begin{prooftree}
\AxiomC{$\neg\bullet$}
\LeftLabel{$I\bullet$}
\UnaryInfC{$\bullet$}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$\bullet$}
\AxiomC{$[\neg\bullet]$}
\noLine
\UnaryInfC{$\vdots$}
\noLine
\UnaryInfC{D}
\LeftLabel{$E\bullet$}
\BinaryInfC{D}
\end{prooftree}
$$

$$
\begin{prooftree}
\AxiomC{$[\bullet]^2$}
\AxiomC{$[\neg\bullet]^1$}
\LeftLabel{$E\bullet_1$}
\BinaryInfC{$\neg\bullet$}
\AxiomC{$[\bullet]^2$}
\LeftLabel{$E\neg$}
\BinaryInfC{$\bot$}
\LeftLabel{$I\neg_2$}
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\AxiomC{$[\bullet]^2$}
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Harmony without validity: relevant conditional

$$\frac{\text{If John is in Paris he is in France} \qquad \text{If John is in London he is in England}}{\text{If John is in Paris he is in England, or if he is in London he is in France.}}$$

$$\frac{A \supset B \qquad C \supset D}{(A \supset D) \vee (C \supset B)}$$

here

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Necessary truth preservation as validity

Normativity of logic: The right logic is an **harmonious** logic that **necessarily preserves truth**.

"Proof-theoretic validity serves an epistemological function to reveal how those inferences result from the meaning-determining rules alone. But it cannot serve the metaphysical function of actually making those inferences valid. Validity is truth preservation, and proof must respect that fact."

Read rejects Tarski's account of necessary truth preservation: **models** and **truth tables**.

"Nonetheless, this is not to equate validity with preservation of truth through arbitrary replacement of the non-logical vocabulary. [. . .] Validity is necessary truth preservation, in itself dependent on the meanings of the constituent propositions."

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Objection

So we are back at our starting point: how can Read and another proof-theoretic semanticist disagree?

$A \supset B, C \supset D \vdash (A \supset D) \vee (C \supset B)$ preserves truth?

There is no way out Quine puzzle!

If we ask if $A \supset B, C \supset D \vdash (A \supset D) \vee (C \supset B)$ preserves truth *simpliciter*, we can at most discover which conditional is applied in natural language.

A purely descriptive problem!

If we know the rules of a logic, we can discover if $A \supset B, C \supset D \vdash (A \supset D) \vee (C \supset B)$ is valid in it. (And truth? here)

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More

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Routley's generalization of Kripke's semantics: given the worlds x,y and z, $x \Vdash A \supset B$ iff for every world y and z, if z is accessible from y according to x, then if $y \Vdash A$, $z \Vdash B$.

In this way we could save the **realist solution for relevant logic**, but not an antirealist solution!

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Proof-theoretic consequence is based on preservation of grounds for assertion.

Grounds for assertion replace truth!

So, for classical logic $A \supset B, C \supset D \vdash (A \supset D) \vee (C \supset B)$ preserves “truth”, that is grounds for assertion.

Definition of validity in $\mathscr{B}$ (Prawitz)
A derivation $\mathfrak{D}$ is valid in $\mathscr{B}$ iff either:

1. $\mathfrak{D}$ is a closed derivation of an atomic conclusion C and it can be reduced by normalization to a closed proof of the same conclusion C carried on in $\mathscr{B}$; or
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3. $\mathfrak{D}$ is an open derivation and every closure of $\mathfrak{D}$, obtained by replacing open assumptions by closed derivations for the same sentences that are valid in $\mathscr{B}$, is valid in $\mathscr{B}$.

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