Is logical disagreement possible in inferentialism?

Leonardo Ceragioli

Università di Pisa e Firenze

May 30, 2019



UNIVERSITÀ DI PISA

Contact: leonardo.ceragioli@unifi.it





イロト イヨト イヨト イ

The puzzle of logical disagreement

2 A realist solution

3 An antirealist solution

- Standard proof-theoretic semantics
- Proof-theoretic semantics for classical logic
- The debate

(日)

The puzzle of logical disagreement

・ロト ・日 ・ ・ ヨ ・ ・

Identity and disagreement

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

Alice: Mum is blonde! Bob: Mum has brown hair!

Bob: Mum has brown hair!

They disagree if and only if they are brother and sister.

Bob: Mum has brown hair!

They disagree if and only if they are brother and sister. Why?

Bob: Mum has brown hair!

They disagree if and only if they are brother and sister. Why? Because otherwise they do not speak of the same person!

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

 $\mathsf{Classical \ logic:} \, \vDash p \lor \neg p$

Classical logic: $\vDash p \lor \neg p$ Intuitionistic logic: $\nvDash p \lor \neg p$

Classical logic: $\vDash p \lor \neg p$ Intuitionistic logic: $\nvDash p \lor \neg p$

In order to disagree, two logics have to speak about the same logical terms.

Classical logic: $\vDash p \lor \neg p$ Intuitionistic logic: $\nvDash p \lor \neg p$

In order to disagree, two logics have to speak about the same logical terms.

They disagree

They don't disagree

・ロト ・ 四ト ・ ヨト ・ ヨト

Realist semantics: the meaning of a logical term is given by **truth conditions** of sentences that have it as principal operator: **truth tables give meaning**.

Realist semantics: the meaning of a logical term is given by **truth conditions** of sentences that have it as principal operator: **truth tables give meaning**. Antirealist semantics: the meaning of a logical term is given by **assertion conditions** of sentences that have it as principal operator: **inference rules give meaning**.

Realist semantics: the meaning of a logical term is given by **truth conditions** of sentences that have it as principal operator: **truth tables give meaning**. Antirealist semantics: the meaning of a logical term is given by **assertion conditions** of sentences that have it as principal operator: **inference rules give meaning**.

Both truth tables and rules of inference determine univocally the class of logical laws for a class of logical terms.

Realist semantics: the meaning of a logical term is given by **truth conditions** of sentences that have it as principal operator: **truth tables give meaning**. Antirealist semantics: the meaning of a logical term is given by **assertion conditions** of sentences that have it as principal operator: **inference rules give meaning**.

Both truth tables and rules of inference determine univocally the class of logical laws for a class of logical terms.

In order to be the same, two logical terms should have the same truth tables (according to realism) or they should have the same rules (according to antirealism). So **if they are the same, they validate the same logical laws.**

The puzzle:

(日)

() In order to disagree, two logics have to speak about the same logical terms;

() In order to disagree, two logics have to speak about the same logical terms;

If two logical terms are the same then they validate the same logical laws.

- In order to disagree, two logics have to speak about the same logical terms;
- **(a)** If two logical terms are the same then they validate the same logical laws.

If two logical terms validate different logical laws, then they are different.

If two logical terms validate different logical laws, then they are different. If two terms are different, then the two logics do not disagree because of them.

If two logical terms validate different logical laws, then they are different. If two terms are different, then the two logics do not disagree because of them.

So logical disagreement seems to be impossible!

If two logical terms validate different logical laws, then they are different. If two terms are different, then the two logics do not disagree because of them.

So logical disagreement seems to be impossible!

'Change of logic, change of subject.' (Quine, *Philosophy of Logic*, 1986, chapter *Deviant logics*.)

If two logical terms validate different logical laws, then they are different. If two terms are different, then the two logics do not disagree because of them.

So logical disagreement seems to be impossible!

'Change of logic, change of subject.' (Quine, *Philosophy of Logic*, 1986, chapter *Deviant logics*.)

- $\bullet \models_k p \lor_k \neg_k p$
- $\bullet \nvDash_i p \vee_i \neg_i p$

A realist solution

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

■ ► 重 ∽ ९ (~ May 30, 2019 8/33

Meaning and behaviour in realism

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

Meaning and behaviour in realism

$A \vee B$	T	F
Т	Т	Т
F	Т	F



< ロ ト < 回 ト < 三 ト < 三 ト</p>

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?



<ロト <回ト < 回ト < 回ト -



If we assume **bivalence** (every sentence is true or false) they lead to **classical logic**.

<ロト <回ト < 回ト < 回ト -



If we assume **bivalence** (every sentence is true or false) they lead to **classical logic**.

BUT if we do not assume bivalence they they can characterise trivalent connectives in an incomplete way!



If we assume **bivalence** (every sentence is true or false) they lead to **classical logic**.

BUT if we do not assume bivalence they they can characterise trivalent connectives in an incomplete way!

They also suit intuitionistic connectives if we do not accept bivalence. (McDowell, Meaning, bivalence, and verificationism, in Gareth Evans ed., Truth and meaning: essays in semantics, 1976, pp. 42-66.)

< = > < @ > < E > < E > < E</p>

▲□▶ ▲圖▶ ▲臣▶ ▲臣▶



ヘロト ヘロト ヘヨト ヘヨト

	$\neg A$	$A \lor B$	Т	U	F
Т	F	Т	Т	Т	Т
U	U	U	Т	U	U
F	Т	F	Т	U	F

$$\Gamma \vDash_{K3} \Delta \ sse \ \bar{\forall} v((\bar{\forall} \gamma_{\in \Gamma} v(\gamma) = T) \Rightarrow (\bar{\exists} \delta_{\in \Delta} v(\delta) = T))$$

designated value: T

(日)



 $\Gamma \vDash_{K3} \Delta \ sse \ \bar{\forall} v((\bar{\forall} \gamma_{\in \Gamma} v(\gamma) = T) \Rightarrow (\bar{\exists} \delta_{\in \Delta} v(\delta) = T)) \\ \qquad \qquad \text{designated value: } \mathsf{T}$

If we assume gaps in truth values, they lead to Kleene's logic.



 $\Gamma \vDash_{K3} \Delta \ sse \ \bar{\forall} v((\bar{\forall} \gamma_{\in \Gamma} v(\gamma) = T) \Rightarrow (\bar{\exists} \delta_{\in \Delta} v(\delta) = T)) \\ \qquad \qquad \text{designated value: } \mathsf{T}$

If we assume gaps in truth values, they lead to Kleene's logic.

If we assume **bivalence**, they lead to **classical logic**.



 $\Gamma \vDash_{K3} \Delta \ sse \ \bar{\forall} v((\bar{\forall} \gamma_{\in \Gamma} v(\gamma) = T) \Rightarrow (\bar{\exists} \delta_{\in \Delta} v(\delta) = T)) \\ \qquad \qquad \text{designated value: } \mathsf{T}$

If we assume gaps in truth values, they lead to Kleene's logic.

If we assume **bivalence**, they lead to **classical logic**.

Idea: K is essentially K3 where there are no gaps!

Beall & Restall solution

The difference between \vee_k and \vee_{k3} is not a difference in truth tables, so they have the same meaning.

Realist meaning + metaphysical shape of the models \rightarrow behaviour.

Realist meaning + metaphysical shape of the models \rightarrow behaviour.

In general:

Realist meaning + metaphysical shape of the models \rightarrow behaviour.

In general:

Generalized Tarski Thesis (GTT): An argument is valid_x if and only if, in every case_x in which the premises are true, so is the conclusion.(Beall & Restall, *Logical Pluralism*, 2006, p. 29.)

イロト イロト イヨト イヨト

Realist meaning + metaphysical shape of the models \rightarrow behaviour.

In general:

Generalized Tarski Thesis (GTT): An argument is valid_x if and only if, in every $case_x$ in which the premises are true, so is the conclusion.(Beall & Restall, *Logical Pluralism*, 2006, p. 29.)

Different sets of cases detect different logics:

イロト イボト イヨト イヨト

Realist meaning + metaphysical shape of the models \rightarrow behaviour.

In general:

Generalized Tarski Thesis (GTT): An argument is valid_x if and only if, in every $case_x$ in which the premises are true, so is the conclusion.(Beall & Restall, *Logical Pluralism*, 2006, p. 29.)

Different sets of cases detect different logics: cases can have gaps in truth values, gluts in truth values, etc.

イロト イボト イヨト イヨト

Realist meaning + metaphysical shape of the models \rightarrow behaviour.

In general:

Generalized Tarski Thesis (GTT): An argument is valid_x if and only if, in every $case_x$ in which the premises are true, so is the conclusion.(Beall & Restall, *Logical Pluralism*, 2006, p. 29.)

Different sets of cases detect different logics: cases can have gaps in truth values, gluts in truth values, etc.

A logical realist can reject the point 2 of the puzzle: two logical terms can be the same also if they validate different logical laws.

<ロト < 回 > < 回 > < 回 > < 回 >

An antirealist solution

イロト イロト イヨト イヨト

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

イロト イロト イヨト イヨト

So a difference in inferential behaviour is a difference in meaning.

イロト イロト イヨト イヨト

So a difference in inferential behaviour is a difference in meaning.

So we can not reject point 2 of the puzzle!

So a difference in inferential behaviour is a difference in meaning.

So we can not reject point 2 of the puzzle!

We need another solution!

イロト イロト イヨト イヨト

イロト イロト イモト イモト

Prawitz & Dummett: tertium non datur does not hold!

Prawitz & Dummett: *tertium non datur* does not hold! Boričić, Read & Milne: *tertium non datur* holds!

Prawitz & Dummett: *tertium non datur* does not hold! Boričić, Read & Milne: *tertium non datur* holds!

(a rational reconstruction of the debate)

Prawitz & Dummett: *tertium non datur* does not hold! Boričić, Read & Milne: *tertium non datur* holds!

(a rational reconstruction of the debate)

We will see that **their disagreement is not only apparent!** They don't talk past each other.

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

• Introduction rules are meaning conferring;

- Introduction rules are meaning conferring;
- Elimination rules are justified by I-rules.

- Introduction rules are meaning conferring;
- Elimination rules are justified by I-rules.

"The introductions represent, as it were, the 'definitions' of the symbols concerned, and the eliminations are no more, in the final analysis, than the consequences of these definitions. This fact may be expressed as follows: In eliminating a symbol, we may use the formula with whose terminal symbol we are dealing only 'in the sense afforded it by the introduction of that symbol'." (Gentzen, *Investigation into Logical Deduction*, 1934/35, 5.13)

イロト イボト イヨト イヨト

Harmony

<ロト < 回 ト < 臣 ト < 臣 ト</p>

イロト イロト イヨト イヨト

• E-rules can not be derived from I-rules;

- E-rules can not be derived from I-rules;
- E-rules do not outstrip the l-rules if, when we have the major premise of an E-rule derived using an l-rule, then we have an avoidable detour;

- E-rules can not be derived from I-rules;
- E-rules do not outstrip the l-rules if, when we have the major premise of an E-rule derived using an l-rule, then we have an avoidable detour;
- Otherwise the E-rules are not justified!

イロト イロト イヨト イヨト

$$\frac{A}{A \lor B} \lor I$$

$$\frac{B}{A \lor B} \lor I$$

$$[A]$$

$$\vdots$$

$$\frac{B}{A \supset B} \supset I$$

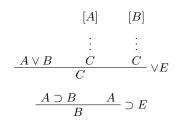
$$\frac{A}{A \lor B} \lor I$$

$$\frac{B}{A \lor B} \lor I$$

$$[A]$$

$$\vdots$$

$$\frac{B}{A \supset B} \supset I$$



<ロト < 回ト < 三ト < 三ト</p>

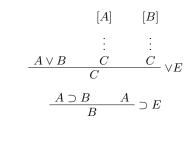
$$\frac{A}{A \lor B} \lor I$$

$$\frac{B}{A \lor B} \lor I$$

$$[A]$$

$$\vdots$$

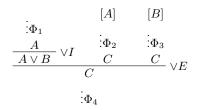
$$\frac{B}{A \supset B} \supset I$$

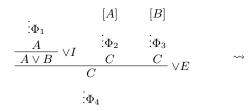


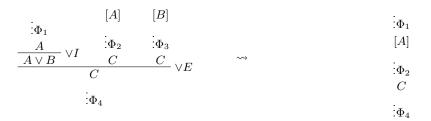
<ロト <回ト < 回ト < 回ト -

 $\frac{\perp}{A} \perp E$

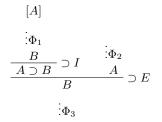
Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

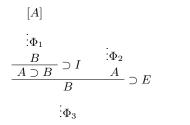






Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?





 $\sim \rightarrow$



▲口 → ▲圖 → ▲臣 → ▲臣 →

 $Itonk \frac{A}{AtonkB} \quad Etonk \frac{AtonkB}{B}$

イロト イヨト イヨト イヨト 二日

$$Itonk \frac{A}{AtonkB} \quad Etonk \frac{AtonkB}{B}$$
$$Itonk \frac{A}{Etonk} \frac{A}{B}$$

▲口 → ▲圖 → ▲臣 → ▲臣 →

$$Itonk \frac{A}{AtonkB} \quad Etonk \frac{AtonkB}{B}$$
$$Itonk \frac{A}{Etonk} \frac{A}{B} \quad \rightsquigarrow$$

900

◆□▶ ◆圖▶ ◆臣▶ ◆臣▶

$$Itonk \frac{A}{AtonkB} \quad Etonk \frac{AtonkB}{B}$$
$$\frac{Itonk}{Etonk} \frac{A}{AtonkB} \quad \rightsquigarrow \quad ?$$

◆□▶ ◆圖▶ ◆臣▶ ◆臣▶

$$Itonk \frac{A}{AtonkB} \quad Etonk \frac{AtonkB}{B}$$
$$Itonk \frac{A}{Etonk} \frac{A}{B} \quad \rightsquigarrow \quad ?$$

• *Tonk* is not an harmonious connective, and indeed it leads to triviality in standard logical systems.

$$Itonk \frac{A}{AtonkB} \quad Etonk \frac{AtonkB}{B}$$
$$\frac{Itonk}{Etonk} \frac{A}{AtonkB} \quad \rightsquigarrow \quad ?$$

• *Tonk* is not an harmonious connective, and indeed it leads to triviality in standard logical systems.

(Prior, the runabout inference ticket, Analysis (21), 1961, pp. 38-39)

Rejection of tertium non datur

The standard harmonious rules are complete for intuitionistic logic, but not for classical logic.

The standard harmonious rules are complete for intuitionistic logic, but not for classical logic.

Conjecture (Prawitz & Dummett): We can not prove *tertium non datur* using harmonious rules!

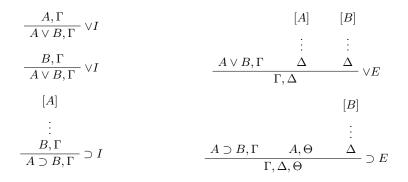
The standard harmonious rules are complete for intuitionistic logic, but not for classical logic.

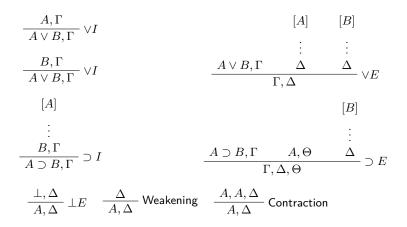
Conjecture (Prawitz & Dummett): We can not prove *tertium non datur* using harmonious rules!

(Prawitz, Towards a foundation of general proof theory, in P. Suppes et al (ed), Logic, Methodology and Philosophy of Science IV, 1973, pp. 225-50.)

(Dummett, The Logical Basis of Metaphysics, 1991.)

$$\begin{array}{c} A, \Gamma \\ \hline A \lor B, \Gamma \\ \hline \end{array} \lor I \\ \hline \hline B, \Gamma \\ \hline A \lor B, \Gamma \\ \hline \blacksquare \\ I \\ \hline B, \Gamma \\ \hline A \supset B, \Gamma \\ \hline \supset I \end{array}$$





<ロト <回ト < 三ト < 三ト -

Acceptance of tertium non datur

The rules are **harmonious** and **complete for classical logic** (consider **Cut** elimination for sequent calculus **LK**).

The rules are **harmonious** and **complete for classical logic** (consider **Cut** elimination for sequent calculus **LK**).

Boričić & Read: We can prove tertium non datur using harmonious rules!

The rules are **harmonious** and **complete for classical logic** (consider **Cut** elimination for sequent calculus **LK**).

Boričić & Read: We can prove tertium non datur using harmonious rules!

 $\frac{ \begin{bmatrix} A \end{bmatrix}^1}{A, \bot}$ Weakening $\frac{A, \bot}{A, A \supset \bot} \supset I_1$

<ロト <回ト < 注ト < 注ト -

The rules are **harmonious** and **complete for classical logic** (consider **Cut** elimination for sequent calculus **LK**).

Boričić & Read: We can prove tertium non datur using harmonious rules!

 $\frac{ \begin{bmatrix} A \end{bmatrix}^1}{A, \bot}$ Weakening $\frac{A, \bot}{A, A \supset \bot} \supset I_1$

(Boričić, *On sequence-conclusion natural deduction systems*, Journal of Philosophical Logic (14), 1985, pp. 359-377.)

(Read, Harmony and autonomy in classical logic, Journal of Philosophical Logic (29), 2000, pp. 123-54.)

< = > < @ > < E > < E > < E</p>

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

Multiple conclusions are not commonly used!

• Multiple conclusions are not commonly used!

[...] the rarity, to the point of extinction, of naturally occurring multipleconclusion arguments has always been the reason why mainstream logicians have dismissed multiple-conclusion logic as little more than a curiosity. (Rumfitt, Knowledge by deduction, Grazer Philosophische Studien (77), 2008, pp. 61-84)

• Multiple conclusions are not commonly used!

[...] the rarity, to the point of extinction, of naturally occurring multipleconclusion arguments has always been the reason why mainstream logicians have dismissed multiple-conclusion logic as little more than a curiosity. (Rumfitt, Knowledge by deduction, Grazer Philosophische Studien (77), 2008, pp. 61-84)

• Multiple conclusions lead to non-constructivity!

Multiple conclusions are not commonly used!

[...] the rarity, to the point of extinction, of naturally occurring multipleconclusion arguments has always been the reason why mainstream logicians have dismissed multiple-conclusion logic as little more than a curiosity. (Rumfitt, Knowledge by deduction, Grazer Philosophische Studien (77), 2008, pp. 61-84)

• Multiple conclusions lead to non-constructivity!

 $[\ldots]$ this smuggles in non-constructivity through the back door. (Tennant, The Taming of the True, 1997)

イロト イヨト イヨト

Multiple conclusions are not commonly used!

[...] the rarity, to the point of extinction, of naturally occurring multipleconclusion arguments has always been the reason why mainstream logicians have dismissed multiple-conclusion logic as little more than a curiosity. (Rumfitt, Knowledge by deduction, Grazer Philosophische Studien (77), 2008, pp. 61-84)

• Multiple conclusions lead to non-constructivity!

 $[\ldots]$ this smuggles in non-constructivity through the back door. (Tennant, The Taming of the True, 1997)

• Multiple conclusions are just disjunction in disguise!

イロト 不得 トイヨト イヨト

• Multiple conclusions are not commonly used!

[...] the rarity, to the point of extinction, of naturally occurring multipleconclusion arguments has always been the reason why mainstream logicians have dismissed multiple-conclusion logic as little more than a curiosity. (Rumfitt, Knowledge by deduction, Grazer Philosophische Studien (77), 2008, pp. 61-84)

• Multiple conclusions lead to non-constructivity!

 $[\ldots]$ this smuggles in non-constructivity through the back door. (Tennant, The Taming of the True, 1997)

• Multiple conclusions are just disjunction in disguise!

[...] in a succedent comprising more than one sentence, the sentences are connected disjunctively; and it is not possible to grasp the sense of such a connection otherwise than by learning the meaning of the constant 'or'. (Dummett, Ibidem, p. 187)

<ロト <回ト < 三ト < 三ト -

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

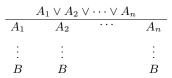
Disagreements: common usages of multiple conclusions.

According to Shoesmith, Smiley and Restall a common usage of multiple conclusions is **proof by cases**:

$A_1 \lor A_2 \lor \cdots \lor A_n$				
A_1	A_2	• • •	A_n	
B	B		B	

Disagreements: common usages of multiple conclusions.

According to Shoesmith, Smiley and Restall a common usage of multiple conclusions is **proof by cases**:



Rumfitt and Steinberger disagree, and consider some single-conclusion formulations of the same proof.

・ロト ・回ト ・ヨト・

Disagreements: common usages of multiple conclusions.

According to Shoesmith, Smiley and Restall a common usage of multiple conclusions is **proof by cases**:

$A_1 \lor A_2 \lor \cdots \lor A_n$				
A_1	A_2	•••	A_n	
B	B		B	

Rumfitt and Steinberger disagree, and consider some single-conclusion formulations of the same proof.

Shoesmith & Smiley, Multiple Conclusion Logic, 1978.

Restall, Multiple conclusions, in Petr Hajek ed., Logic, Methodology and Philosophy of Science: Proceedings of the Twelfth International Congress, 2004, pp. 189-205.

Rumfitt, Ibidem.

Steinberger, Why conclusions should remain single, Journal of Philosophical Logic (40), 2011, pp. 333-355.

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

A theory of meaning should use a constructive logic? This is so controversial, I will neglect discussing it!

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

Disagreements: multiple conclusions and disjunction.

Milne's formulates classical logic with:

$$[A] \qquad [A]$$

$$\vdots \qquad \vdots$$

$$I \supset_{Mln} \frac{B\{\lor D\}}{(A \supset B)\{\lor D\}} \quad I \neg_{Mln} \frac{D}{\neg A \lor D}$$

Milne's formulates classical logic with:

$$[A] \qquad [A]$$

$$\vdots \qquad \vdots$$

$$I \supset_{Mln} \frac{B\{\lor D\}}{(A \supset B)\{\lor D\}} \quad I \neg_{Mln} \frac{D}{\neg A \lor D}$$

If **harmony** holds for this formulation of classical logic, then the identification of multiple conclusions with disjunction is not problematic!

Milne's formulates classical logic with:

$$[A] \qquad [A]$$

$$\vdots \qquad \vdots$$

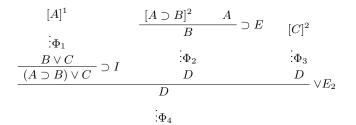
$$I \supset_{Mln} \frac{B\{\lor D\}}{(A \supset B)\{\lor D\}} \quad I \neg_{Mln} \frac{D}{\neg A \lor D}$$

If **harmony** holds for this formulation of classical logic, then the identification of multiple conclusions with disjunction is not problematic!

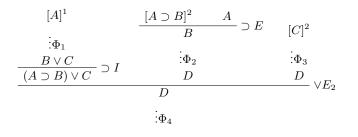
(Milne, Harmony, Purity, Simplicity and a "Seemingly Magical Fact", The Monist (85), 2002, pp. 498-534)



Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

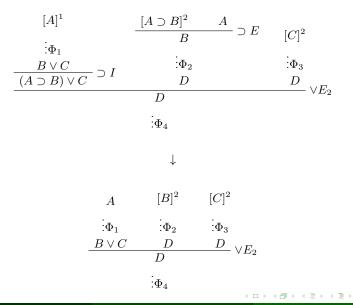


<ロト < 回ト < 三ト < 三ト</p>



 \downarrow

<ロト < 回ト < 三ト < 三ト</p>



Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

Can it be non-constructive?

Can it be non-constructive?

Can it have basic proof steps that are not commonly used?

イロト イボト イヨト イヨ

Can it be non-constructive?

Can it have basic proof steps that are not commonly used?

Essentially: Can it have multiple conclusions?

Conclusion

From an antirealistic point of view, two logics can not disagree, since they speak of different logical terms.

From an antirealistic point of view, two logics can not disagree, since they speak of different logical terms.

Nonetheless, two logicians can disagree about the existence of a good theory of meaning for a logical term.

From an antirealistic point of view, two logics can not disagree, since they speak of different logical terms.

Nonetheless, two logicians can disagree about the existence of a good theory of meaning for a logical term.

Prawitz & Dummett: Every theory of meaning for classical logic is not harmonious, so the meaning of \neg_k is not well defined;

Boričić, Read & Milne: There is a good theory of meaning for classical logic, so $\models p \lor_k \neg_k p.$

<ロト < 回ト < 三ト < 三ト</p>

From an antirealistic point of view, two logics can not disagree, since they speak of different logical terms.

Nonetheless, two logicians can disagree about the existence of a good theory of meaning for a logical term.

Prawitz & Dummett: Every theory of meaning for classical logic is not harmonious, so the meaning of \neg_k is not well defined;

Boričić, Read & Milne: There is a good theory of meaning for classical logic, so $\models p \lor_k \neg_k p.$

They don't talk past each other.

<ロト <回ト < 三ト < 三ト

Leonardo Ceragioli (Università di Pisa e Firenze) Is logical disagreement possible in inferentialism?

▲ 트 ► 트 ∽ ९ (~ May 30, 2019 32 / 33

《日》《圖》《臣》《臣》

Thanks for your attention!

Thanks for your attention!

Bibliography

- Beall & Restall, Logical Pluralism, 2006.
- Boričić, *On sequence-conclusion natural deduction systems*, Journal of Philosophical Logic (14), 1985, pp. 359-377.
- Dummett, The Logical Basis of Metaphysics, 1991.
- Gentzen, Investigation into Logical Deduction, 1934/35, in ed. Szabo, The collected papers of Gerhard Gentzen, 1969.
- McDowell, Meaning, bivalence, and verificationism, in Gareth Evans ed., Truth and meaning: essays in semantics, 1976, pp. 42-66.
- Milne, Harmony, Purity, Simplicity and a "Seemingly Magical Fact", The Monist (85), 2002, pp. 498-534.
- Prior, the runabout inference ticket, Analysis (21), 1961, pp. 38-39.
- Quine, Philosophy of Logic, 1986, chapter Deviant logics.
- Read, *Harmony and autonomy in classical logic*, Journal of Philosophical Logic (29), 2000, pp. 123-54.
- Restall, Multiple conclusions, in Petr Hajek ed., Logic, Methodology and Philosophy of Science: Proceedings of the Twelfth International Congress, 2004, pp. 189-205.
- Rumfitt, Knowledge by deduction, Grazer Philosophische Studien (77), 2008, pp. 61-84.
- Shoesmith & Smiley, Multiple Conclusion Logic, 1978.
- Steinberger, *Why conclusions should remain single*, Journal of Philosophical Logic (40), 2011, pp. 333-355.
- Tennant, The Taming of the True, 1997.