Meaning-Dependence and Weak Separability in Bilateral Systems

Bilateralism and Proof-Theoretic Semantics, Ruhr University Bochum

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Open Issues for Rumfitt's style bilateralism

Observation

*Rumfitt's bilateral system (**Rumfitt**) consist of:*

Operational Rules (OR): rules governing the introduction and elimination of connectives inside an asserted (sign $+$) or rejected (sign $-$) proposition;

$$\wedge I^{+}\ \frac{+A \qquad +B}{+(A \wedge B)}$$

Coordination Principles (CP): principles dealing with assertions and rejections of propositions regardless of the logical structure of the propositions.

$$\text{non-contradiction}\ \frac{+A \qquad -A}{\bot}$$

Observation

OR are acceptable iff they suit harmony: $\oplus^{+}I$ is in harmony with $\oplus^{+}E$, and $\oplus^{-}I$ is in harmony with $\oplus^{-}E$.

Question

1. *Which criterion for the acceptability of CP?*
2. *The choice of $\oplus^{-}$-rules is independent of $\oplus^{+}$-rules?*
3. *Why tonk-rules seem to lead to reducible maximal formulae?*

Which *criterion* for CP?

Which *criterion* for the acceptability of CP?

Observation

First Proposal: *CP should be admissible in the system (implicit in [Rumfitt, 2000]).*

CP were included in the system to make some OP derivable, and so make the system more manageable. They were not intended to be necessary for the system.

Objection

Tertium non datur doesn't hold in ***Rumfitt*** *without CP ([Gibbard, 2002]).*

Observation

Second Proposal: *it should be possible to restrict CP to atomic sentences ([Rumfitt, 2000]).*

Intuitively, if this desideratum were satisfied, CP would not influence the meaning of the connectives, but they would only determine the relation between atomic assertions and atomic rejections.

Objection

CP cannot be restricted to atomic applications ([Ferreira, 2008]).

Observation

We can have bilateral systems for both classical and intuitionistic logic and the main difference between them is due to CP ([Kuřbis, 2016]).

$$\neg\mathsf{I}^{+}\ \frac{-A}{+(\neg A)} \qquad \neg\mathsf{E}^{+}\ \frac{+(\neg A)}{-A} \qquad \neg\mathsf{I}^{-}\ \frac{\begin{array}{c}[-A]\\ \vdots\\ \alpha\end{array} \qquad \begin{array}{c}[-A]\\ \vdots\\ \alpha^*\end{array}}{-(\neg A)} \qquad \neg\mathsf{E}^{-}\ \frac{-(\neg A) \qquad -A}{\beta}$$

Observation

With Rumfitt's CP is classical, with Kuřbis' CP is intuitionistic.

$$\text{Intuitionistic Smiley}\ \frac{\begin{array}{c}[+A]\\ \vdots\\ +B\end{array} \qquad \begin{array}{c}[+A]\\ \vdots\\ -B\end{array}}{-A}$$

Conclusion

Not only CP is unjustified, it is also the main responsible for the selection of the logic.

The choice of $\oplus^{-}$-rules is independent of $\oplus^{+}$-rules?

The choice of $\oplus^-$-rules is independent of $\oplus^+$-rules?

Observation

There are pairs of $\oplus^+$ and $\oplus^-$-rules that lead to trivialism with standard CP ([Gabbay, 2017]).

$$\bullet\mathsf{I}^+ \ \frac{+A \qquad -A}{+\bullet} \qquad\qquad \bullet\mathsf{I}^- \ \frac{+A \qquad -A}{-\bullet}$$

$$\bullet\mathsf{E}^+ \ \frac{+\bullet}{+A} \quad \bullet\mathsf{E}^+ \ \frac{+\bullet}{-A} \quad \bullet\mathsf{E}^- \ \frac{-\bullet}{+A} \quad \bullet\mathsf{E}^- \ \frac{-\bullet}{-A}$$

Observation

They are harmonious in bilateral systems, but:

$$\textit{Smiley}_1 \ \frac{\bullet E^+ \ \dfrac{[+\bullet]^1}{+B} \qquad \bullet E^+ \ \dfrac{[+\bullet]^1}{-B}}{\bullet E^- \ \dfrac{-\bullet}{+A}} \qquad \textit{Smiley}_1 \ \frac{\bullet E^+ \ \dfrac{[+\bullet]^1}{+B} \qquad \bullet E^+ \ \dfrac{[+\bullet]^1}{-B}}{\bullet E^- \ \dfrac{-\bullet}{-A}}$$

Observation

[Francez, 2017]:

Coherence: *You should not assert and reject the same sentence;*

Horizontal Balance: *Rules of rejection should be a function of the rules of assertion.*

Objection

- *Given standard CP, Coherence is equivalent to non-trivialism;*
- *Horizontal Balance leads to Coherence* ***only if*** *CP behave well;*
- *It gives no criterion for CP (are the following rules acceptable?).*

Objection

$$\textit{Incoherence}\ \frac{+A}{-A} \qquad \textit{Incoherence}\ \frac{-A}{+A}$$

Why *tonk*-rules seem to lead to reducible maximal formulae?

Why *tonk*-rules seem to lead to reducible maximal formulae?

Observation

$$\dfrac{\dfrac{+A}{+(A tonk B)}\ tonkI}{+B}\ tonkE$$

$$\Downarrow$$

$$\dfrac{\dfrac{+A}{+(A tonk B)}\ tonkI \qquad \dfrac{\dfrac{[+(A tonk B)]^1}{+B}\ tonkE \qquad [-B]^2}{-(A tonk B)}\ Smiley_1}{+B}\ Smiley_2$$

[Gabbay, 2017]

Observation

[Francez, 2018]:

- *The second derivation does not qualify as a real reduction of the first, because it does not solve the problem caused by tonk, but just spreads it out in the derivation;*
- *"I claim that there is no need to show that such uses are illegitimate".*

“I claim that there is no need to show that such uses are illegitimate”

Observation

$$\vee E\,\dfrac{A \vee B \qquad \oplus E\,\dfrac{\oplus I\,\dfrac{\begin{matrix}[A]\\ \vdots\\ C\end{matrix}}{D}}{E} \qquad \oplus E\,\dfrac{\oplus I\,\dfrac{\begin{matrix}[B]\\ \vdots\\ C\end{matrix}}{D}}{E}}{E} \quad \rightsquigarrow \quad \oplus E\,\dfrac{\vee E\,\dfrac{A \vee B \qquad \oplus I\,\dfrac{\begin{matrix}[A]\\ \vdots\\ C\end{matrix}}{D} \qquad \oplus I\,\dfrac{\begin{matrix}[B]\\ \vdots\\ C\end{matrix}}{D}}{D}}{E}$$

Objection

To refute the reduction we need maximal sequences, it is not enough to say that maximal formula is just replicated!

Definition

Let's consider the following weakened version of *tonk*I:

- the premise of *tonk*I must depend on an assumption that is discharged by an application of $\vee$E;
- the minor premise of $\vee$E that depends on the same assumption is the *tonk*-formula or the conclusion of the rule that has this formula as major premise;
- the sub-derivation of the other minor premise must end in the same way as the first.

Observation

$$\vee E\,\dfrac{A \vee B \qquad tonkE\,\dfrac{tonkI\,\dfrac{\begin{array}{c}[A]\\ \vdots\\ C\end{array}}{C tonk D}}{D} \qquad tonkE\,\dfrac{tonkI\,\dfrac{\begin{array}{c}[B]\\ \vdots\\ C\end{array}}{C tonk D}}{D}}{D} \quad \rightsquigarrow \quad tonkE\,\dfrac{\vee E\,\dfrac{A \vee B \qquad tonkI\,\dfrac{\begin{array}{c}[A]\\ \vdots\\ C\end{array}}{C tonk D} \qquad tonkI\,\dfrac{\begin{array}{c}[B]\\ \vdots\\ C\end{array}}{C tonk D}}{C tonk D}}{D}$$

Observation

$$\vee E_2\,\dfrac{\vee I\,\dfrac{\supset I_1\,\dfrac{[A]^1}{A \supset A}}{(A \supset A) \vee \bot} \qquad tonkE\,\dfrac{tonkI\,\dfrac{[A \supset A]^2}{(A \supset A) tonk \bot}}{\bot} \qquad tonkE\,\dfrac{tonkI\,\dfrac{Efq\,\dfrac{[\bot]^2}{A \supset A}}{(A \supset A) tonk \bot}}{\bot}}{\bot}$$

Conclusion

We need a generalization of maximality to deal with Gabbay's "reduction".

Complex harmony and tonk

A generalization of Harmony

Definition

Simplicity: Every logical constant which occurs in a rule, occurs as principal operator.

Observation (Milne, 2002)

*Milne proposes **complex** (not **simple**) rules for classical introduction of* $\supset$ *and* $\neg$*:*

$$\supset I_{Mln}\ \frac{\begin{matrix}[A]\\ \vdots\\ B \vee D\end{matrix}}{(A \supset B) \vee D} \qquad \neg I_{Mln}\ \frac{\begin{matrix}[A]\\ \vdots\\ D\end{matrix}}{\neg A \vee D}$$

Harmony per $\supset_{Mln}$

$$
\cfrac{\cfrac{\begin{matrix}[A]^1 \\ \vdots\,\Phi_1 \\ B \vee C\end{matrix}}{(A \supset B) \vee C}\supset I \qquad \begin{matrix}\cfrac{[A \supset B]^2 \qquad A}{B}\supset E \\ \vdots\,\Phi_2 \\ D\end{matrix} \qquad \begin{matrix}[C]^2 \\ \vdots\,\Phi_3 \\ D\end{matrix}}{D}\vee E_2
$$

$$
\vdots\,\Phi_4
$$

$$
\downarrow
$$

$$
\cfrac{\begin{matrix}A \\ \vdots\,\Phi_1 \\ B \vee C\end{matrix} \qquad \begin{matrix}[B]^2 \\ \vdots\,\Phi_2 \\ D\end{matrix} \qquad \begin{matrix}[C]^2 \\ \vdots\,\Phi_3 \\ D\end{matrix}}{D}\vee E_2
$$

$$
\vdots\,\Phi_4
$$

Back to main.

Observation

*With **complex** (not **simple**) rules we must change the definition of maximal formula: **complex maximality**.*

$$I \supset_{Mln} \frac{\begin{matrix}[A] \\ \vdots \\ B \vee D\end{matrix}}{(A \supset B) \vee D} \qquad \supset E \; \frac{(A \supset B) \wedge (D \wedge E) \quad \begin{matrix}[E] \\ \vdots \\ A \vee F\end{matrix} \quad \begin{matrix}[B \wedge D] \\ \vdots \\ C\end{matrix}}{C \vee F}$$

Observation

- *I-rules that introduce the connective in subordinate position;*
  - *Use the meaning of other connectives to give meaning to the introduced one;*
- *E-rules that eliminate the connective in subordinate position;*
  - *Exploit the meaning of more than one connective.*

here

Generalization of maximality due to I-rules

Observation

An I-rule can introduce a connective that cannot be eliminated immediately, and yet it produces maximality.

$$\textit{tonk}\mathsf{I}\,\frac{A \vee C}{(A\textit{tonk}B) \vee C} \qquad \textit{tonk}\mathsf{E}\,\frac{A\textit{tonk}B}{B}$$

$$\vee\mathsf{E}_2\,\dfrac{\textit{tonk}\mathsf{I}\,\dfrac{A \vee B}{(A\textit{tonk}B) \vee B} \qquad \textit{tonk}\mathsf{E}_1\,\dfrac{[A\textit{tonk}B]^2 \qquad [B]^1}{B} \qquad [B]^2}{B}$$

here

Observation

An E-rule can eliminate a connective that was introduced some steps before, and yet it produces maximality.

$$\dfrac{\dfrac{A}{A\,tonk\,B}\,tonk\mathsf{I} \qquad \text{}}{}$$

$$tonk\mathsf{E}_1\;\dfrac{(A\,tonk\,B) \wedge C \qquad \begin{matrix}[B \wedge C]^1 \\ \vdots \\ D\end{matrix}}{D}$$

$$\dfrac{\wedge\mathsf{I}_1\;\dfrac{A \qquad tonk\mathsf{I}\;\dfrac{[A]^1}{A\,tonk\,B} \qquad [A]^1}{(A\,tonk\,B) \wedge A} \qquad [B \wedge A]^2}{B \wedge A}\;tonk\mathsf{E}_2$$

here

What about Gabbay's reduction?

Observation

We have a simple maximal formula:

$$\dfrac{\dfrac{+A}{+(AtonkB)}\ tonk\mathsf{I}}{+B}\ tonk\mathsf{E}$$

Observation

And we have a complex maximal formula:

$$\dfrac{\dfrac{+A}{+(AtonkB)}\ tonk\mathsf{I} \qquad \dfrac{\dfrac{[+(AtonkB)]^1}{+B}\ tonk\mathsf{E} \qquad [-B]^2}{-(AtonkB)}\ \mathsf{Smiley}_1}{+B}\ \mathsf{Smiley}_2$$

Observation

In Rumfitt's system, there are complex maximal formulae for every pair of I and E-rules:

$$\text{Smiley}_2\,\dfrac{\wedge I^{+}\,\dfrac{+A \qquad +B}{+A \wedge B} \qquad \text{Smiley}_1\,\dfrac{\wedge E^{+}\,\dfrac{[A \wedge B]^1}{+A} \qquad [-A]^2}{-A \wedge B}}{+A}$$

Observation

But they are all reducible to traditional simple maximal formulae:

$$\wedge E^{+}\,\dfrac{\wedge I^{+}\,\dfrac{+A \qquad +B}{+A \wedge B}}{+A}$$

Conclusion

Gabbay's derivations are:

- *part of the general phenomenon of complex maximality;*
- *not reductions of simple maximal formulae;*
- *ineffective just because of the specific form of* ***CP****.*

Weak Separability and CP

Complex rules and weak separability

Observation

Complex (or just Impure) I-rules impose dependence of meaning between logical terms.

Definition (Weak separability)

To prove a logical consequence $A \vdash B$ we only need to use the rules for the logical constants that occur in A and B, together with the rules for the constants on which those depend.

Example

In Milne's rule, the meaning of $\supset$ depends on that of $\vee$. As a consequence, weak separability allows $\vee$-rules together with $\supset$-rules to derive purely implicational results, like:

$$\cfrac{\cfrac{\cfrac{\cfrac{[p]^1}{p \vee q}\,\vee\mathsf{I}}{p \vee (p \supset q)}\,\supset\mathsf{I}_1 \qquad \cfrac{[(p \supset q) \supset p]^4 \qquad [p \supset q]^2}{p}\,\supset\mathsf{E} \qquad [p]^3}{p}\,\vee\mathsf{E}_{2,3}}{((p \supset q) \supset p) \supset p}\,\supset\mathsf{I}_4$$

Separability regards $+$ and $-$?

Observation

Rumfitt claims that his system is separable, because he considers neither the occurrence of $+$ and $-$, nor the applications of CP.

Observation

Since:

- *CP influence the logic;*
- *Horizontal balance (or something similar) seems needed to restrict $+$ and $-$-rules.*

$+$ and $-$ should be considered for separability as well.

Example

Indeed, let us consider the derivation for $+\neg\neg p \vdash +p$:

$$\neg E^{-}\dfrac{\neg E^{+}\dfrac{+(\neg\neg A)}{-(\neg A)}}{+A}$$

- $\neg$ is the only connective that occurs in it;
- While both assumption and conclusion are assertions, we have to pass through rejection in order to prove the result.

Observation

*Given $\neg I^{+}$, the usage of $-$-rules in the derivation of $+\neg\neg p \vdash +p$ is acceptable by **weak separability**.*

$$\neg I^{+}\dfrac{-A}{+(\neg A)}$$

Further problems: weak separability and CP

Objection

*The rules for $\supset^+$ are the same as intuitionistic **NJ**, but Peirce's law is purely classical: we need rejection in order to prove it, even though it is a purely positive theorem and $\supset I^+$ doesn't use rejection.*

$$\dfrac{\begin{matrix}[+A] \\ \vdots \\ +B\end{matrix}}{+(A \supset B)}\,\supset I^+ \qquad \dfrac{+(A \supset B) \qquad +A}{+B}\,\supset E^+$$

Observation

If we consider CP as I and E-rules for $+$ and $-$, the meanings of those terms depend on each other.

$$\dfrac{\begin{matrix}[+A] \\ \vdots \\ \bot\end{matrix}}{-A}\,-I/+E \qquad \dfrac{\begin{matrix}[-A] \\ \vdots \\ \bot\end{matrix}}{+A}\,+I/-E \qquad \dfrac{+A \qquad -A}{\bot}\,+/-E$$

Are they in harmony? (Postponed to the next section)

Weak separability of Rumfitt's system

Theorem

Weak separability holds for Rumfitt's system.

Proof.

The part about the connectives is given by Rumfitt. About $+$ and $-$, since they depends on each other, any derivation can use rules for the connectives that occur in the premises or in the conclusion of the consequence, signed with the signs that occur in the consequence, together with *Coordination Principles*. Let us just observe that for every connective $\oplus$, $\oplus^+$-rules ($\oplus^-$-rules) are derivable from $\oplus^-$-rules ($\oplus^+$-rules) together with *Coordination Principles*, and the result follows. □

Tentative solution to Gabbay's •

Gabbay's diagnosis for Read's •

Observation

Read rule •I ([Read, 2000]) is also an E-rule for negation, which violates harmony:

$$\bullet I\ \frac{\neg\bullet}{\bullet} \qquad \bullet E\ \frac{\bullet \qquad \begin{matrix}[\neg\bullet]\\ \vdots \\ C\end{matrix}}{C}$$

[Gabbay, 2017]

Observation

$\bullet I^{+}$ and $\bullet E^{-}$ are also E-rules for $-$, $\bullet I^{-}$ and $\bullet E^{+}$ are also E-rules for $+$, $\bullet E^{+}$ is also an I-rule for $-$ and $\bullet E^{-}$ is also an I-rule for $+$.

$$\bullet I^{+}/{-E}\ \frac{+A \qquad -A}{+\bullet} \qquad \bullet I^{-}/{+E}\ \frac{+A \qquad -A}{-\bullet}$$

$$\bullet E^{+}\ \frac{+\bullet}{+A} \quad \bullet E^{+}/{-I}/{+E}\ \frac{+\bullet}{-A} \quad \bullet E^{-}/{+I}/{-E}\ \frac{-\bullet}{+A} \quad \bullet E^{-}\ \frac{-\bullet}{-A}$$

Disharmony of Gabbay's • with standard CP

Observation

Gabbay's rules are not really harmonious, if considered together with standard CP:

$$\dfrac{-I\,\dfrac{+\bullet}{-A} \qquad +A}{\bot}\,-E$$

Observation

The same situation holds regarding the rules

$$\textit{Incoherence}\ \frac{+A}{-A} \qquad \textit{Incoherence}\ \frac{-A}{+A}$$

which are incoherent only if endorsed together with standard CP. There is nothing specially wrong about them, contra Francez.

Conclusion

So, considering + and − for separability and harmony enables:

- *to exclude Gabbay's rules for • inside the usual Rumfitt's system;*
- *to evaluate CP in general.*

Objection: disharmony of Rumfitt's system

Objection

$\neg I^{+}$ is also an I-rule for $+$, and $\supset E^{-}$ is also an E-rule for $+$. They form together the following maximality, which is irreducible:

$$\cfrac{\cfrac{-A}{+(\neg A)}\;+I \qquad -B}{-(\neg A \supset B)}\;+E$$

So, Rumfitt's system is not in harmony.

Observation

Gabbay's rules for $\bullet$ extend the logical consequences regarding only $+$ and $-$ ($+A \dashv\vdash -A$), while Rumfitt's rules do not (they're not consequences regarding only $+$ and $-$).

Assumption

A rule in which $\oplus$ occurs counts as $\oplus$I or $\oplus$E only if it extends the results purely about $\oplus$.

Example

Milne's I-rule for classical negation is not an I-rule for $\vee$, since purely disjunctive classical theorems coincide with purely disjunctive intuitionistic ones.

$$\dfrac{\begin{matrix}[A] \\ \vdots \\ B\end{matrix}}{\neg A \vee B}\ \neg\mathsf{I}_{Milne}$$

"What we must not do is consider this a new, canonical, meaning-specifying rule for the introduction of disjunction as dominant operator" [Milne, 2002], p. 527.

Observation

Gabbay's rules for $\bullet$ make the system disharmonious, while Rumfitt's rules for $\neg$ and $\supset$ do not.

Summary

Observation

Interpreting bilateral systems as complex systems we can avoid Gabbay's reduction for tonk.

Observation

This asks for a weakening of separability (since $+$ and $-$ become relevant for separability).

Observation

Gabbay's $\bullet$-rules are excluded together with unorthodox CP.

Observation

Standard CP are not excluded neither by harmony, nor by separability.

Thank you!

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Extras

Table: **Operational Rules**

$$\wedge\mathsf{I}^{+}\ \frac{+A \qquad +B}{+(A \wedge B)} \qquad \wedge\mathsf{E}^{+}\ \frac{+(A \wedge B)}{+A} \qquad \wedge\mathsf{E}^{+}\ \frac{+(A \wedge B)}{+B}$$

$$\wedge\mathsf{E}^{-}\ \frac{-(A \wedge B) \qquad \begin{matrix}[-A]\\ \vdots \\ C\end{matrix} \qquad \begin{matrix}[-B]\\ \vdots \\ C\end{matrix}}{C} \qquad \wedge\mathsf{I}^{-}\ \frac{-A}{-(A \wedge B)} \qquad \wedge\mathsf{I}^{-}\ \frac{-B}{-(A \wedge B)}$$

$$\vee\mathsf{E}^{+}\ \frac{+(A \vee B) \qquad \begin{matrix}[+A]\\ \vdots \\ C\end{matrix} \qquad \begin{matrix}[+B]\\ \vdots \\ C\end{matrix}}{C} \qquad \vee\mathsf{I}^{+}\ \frac{+A}{+(A \vee B)} \qquad \vee\mathsf{I}^{+}\ \frac{+B}{+(A \vee B)}$$

$$\vee\mathsf{I}^{-}\ \frac{-A \qquad -B}{-(A \vee B)} \qquad \vee\mathsf{E}^{-}\ \frac{-(A \vee B)}{-A} \qquad \vee\mathsf{E}^{-}\ \frac{-(A \vee B)}{-B}$$

Table: **Operational Rules II**

$$\supset I^{+}\ \frac{\begin{array}{c}[+A]\\ \vdots \\ +B\end{array}}{+(A \supset B)} \qquad \supset E^{+}\ \frac{+(A \supset B) \qquad +A}{+B}$$

$$\supset I^{-}\ \frac{+A \qquad -B}{-(A \supset B)} \qquad \supset E^{-}\ \frac{-(A \supset B)}{+A} \qquad \supset E^{-}\ \frac{-(A \supset B)}{-B}$$

$$\neg I^{+}\ \frac{-A}{+(\neg A)} \qquad \neg E^{+}\ \frac{+(\neg A)}{-A}$$

$$\neg I^{-}\ \frac{+A}{-(\neg A)} \qquad \neg E^{-}\ \frac{-(\neg A)}{+A}$$

Table: **Coordination Principles**

$$\textit{Reductio}\;\dfrac{\begin{matrix}[+A]\\ \vdots\\ \bot\end{matrix}}{-A}\qquad\qquad \textit{Reductio}\;\dfrac{\begin{matrix}[-A]\\ \vdots\\ \bot\end{matrix}}{+A}\qquad\qquad \text{non-contradiction}\;\dfrac{+A \qquad -A}{\bot}$$

$$\text{Smiley}\;\dfrac{\begin{matrix}[+A]\\ \vdots\\ +B\end{matrix}\qquad\begin{matrix}[+A]\\ \vdots\\ -B\end{matrix}}{-A}\qquad\qquad \text{Smiley}\;\dfrac{\begin{matrix}[-A]\\ \vdots\\ +B\end{matrix}\qquad\begin{matrix}[-A]\\ \vdots\\ -B\end{matrix}}{+A}$$

Definition (Dependence of meaning)

$\ominus \prec \oplus$ (the meaning of $\oplus$ depends on the meaning of $\ominus$) iff there is a sequence of logical terms $\circ_1, \ldots, \circ_n$ such that $\circ_1 = \ominus$, $\circ_n = \oplus$ and for every $1 \leqslant i < n$, $\circ_i$ occurs in the premisses or in the discharged assumptions of an I-rule for $\circ_{i+1}$.

Definition (Weak separability)

To prove a logical consequence $A \vdash B$ we only need to use the rules for the logical constants that occur in A and B, together with the rules for the constants on which those depend. That is in order to prove a logical consequence $A \vdash B$, it is enough to use the rules for the constants $\circ_1, \ldots, \circ_n$ such that for every $1 \leqslant i \leqslant n$, $\circ_i$ occurs in A or B, or for some $j \neq i$ such that $1 \leqslant j \leqslant n$, $\circ_j$ occurs in A or B and $\circ_i \prec \circ_j$.

Back to main.

Definition (Maximal formulae (**SASCNJ**))

Given a derivation $\mathfrak{D}$, a formula A that occurs in it is a *maximal formula* iff:

- A is the major premise of an application of a $\oplus$E rule and the last rules applied in its immediate subderivation are I-rules for all the connectives which occur actively in it in that application of $\oplus$E.

Back to main.

Definition (Elimination Path (E-path))

Given a derivation $\mathfrak{D}$, a list of sentences $A_1, \ldots, A_n$ is an E-path iff for every m such that $1 \leqslant m \leqslant n$, A_m is the major premise of an E-rule and A_{m+1} is one of the discharged assumption of that rule.

Definition (Maximal formulae (**SASCNK**))

Given a derivation $\mathfrak{D}$, a formula that occurs in it is a *maximal formula* iff:

1. A is the major premise of an application of a $\oplus$E rule and the last rules applied in its immediate subderivation are I-rules for all the connectives which occur actively in it in that application of $\oplus$E; or
2. it is the conclusion of an application of $\oplus$I and the first formula of an E-path such that:
   1. the last rule of the E-path is $\oplus$E;
   2. each rule in the E-path eliminates occurrences of connectives that are active in the conclusion of the application of $\oplus$I.

Back to main.

Observation

In Rumfitt's classical system, there is a circular dependence of meaning between + and −.

Assumption (Complexity condition (Dummett))

The form of the I-rule should guarantee that, in any application of it, the conclusion will be of higher logical complexity than any of the premises and than any discharged hypothesis.

$$\neg I_{Dummett}\ \frac{\begin{matrix}[A] \\ \vdots \\ B\end{matrix} \qquad \begin{matrix}[A] \\ \vdots \\ \neg B\end{matrix}}{\neg A} \qquad \Box I_T\ \frac{\Box\Gamma \qquad \begin{matrix}[\Gamma] \\ \vdots \\ A\end{matrix}}{\Box A} \qquad \Box I_{S4}\ \frac{\begin{matrix}\Box\Gamma \\ \vdots \\ A\end{matrix}}{\Box A} \qquad \Box I_{S5}\ \frac{\begin{matrix}\Box\Gamma, \neg\Box\Delta \\ \vdots \\ A\end{matrix}}{\Box A}$$

Observation

CP violate complexity condition.

$$\wedge I\ \frac{C \qquad \begin{matrix}[C] \\ \vdots \\ A\end{matrix} \qquad \begin{matrix}[C] \\ \vdots \\ B\end{matrix}}{A \wedge B}$$

Question

Which alternatives to Dummett's complexity condition?

$$N\mathrm{I}\ \frac{Nt}{Nst} \qquad \begin{array}{c}[\bullet]\\ \vdots \\ \bullet\mathrm{I}\ \dfrac{\bot}{\bullet}\end{array} \qquad \lambda\mathrm{I}\ \frac{A[x/t]}{t \in \lambda x A} \qquad Yablo\mathrm{I}\ \frac{S_n}{\forall_{k>n}\neg T(S_k)}$$

here

Assumption (Normalization condition (Prawitz-Tennant))

The I-rule should not produce, together with its harmonious E-rule, a non-terminating reduction sequence.

Objection (Open issues for Prawitz-Tennant)

- *Normalization really excludes paradoxes? Normal Curry's paradox in classical logic (Rogerson, 2007);*
- *Non-normalization really leads to paradox? Ekman: the reduction of $\neg A \to A, A \to \neg A \vdash \bot$ goes in loop (Schroeder-Heister and Tranchini, 2017) ;*
- *Why Ekman's paradox works only for **MPP** and not for $\to E_g$?*
- *Non-locality of normalization.*

Assumption (Positiveness condition (Dyckhoff))

The only occurrences of the defininendum in the I-rule should be strictly positive.

Positive $N = \mu X(1 + X)$

Non-Positive $\bullet = \mu X(X \rightarrow \bot)$; $a \in a \leftrightarrow (a \in a \rightarrow p)$

Objection (Open issues for Dyckhoff)

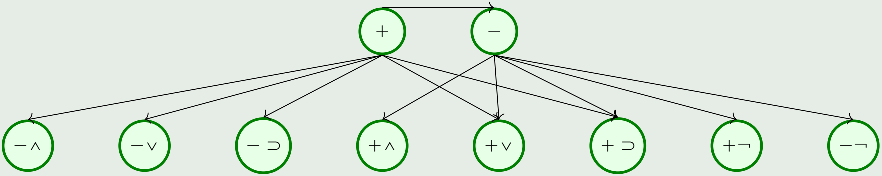
- *Dyckhoff's criterion excludes Curry, (solving Rogerson, 2007);*
- *How Dyckhoff's criterion applies to to Yablo's paradox?*

Separability in Kürbis' intuitionistic system

Observation

In Kürbis' intuitionistic system, rejection depends on assertion but not vice-versa. Moreover, the only assertive I-rule that uses rejection is $\neg I^{+}$.

Observation



Observation

As a consequence, the positive and assertive fragment of the logic should be provable using only assertive rules. This is evident from separability of ***NJ****, since the positive and assertive fragment of Kürbis' system is identical to the positive fragment of* ***NJ****.*